Math 115
Spring 2017
Lecture 14

Ch.4 : Exponential Rules

① $x^n = \underbrace{x \cdot x \cdot x \cdot x \cdot \cdots \cdot x}_{n \text{ times}}$

x base

n Exponent

ex: $x^4 = x \cdot x \cdot x \cdot x$, $(-2x)^3 = (-2x)\cdot(-2x)\cdot(-2x)$

$(3x^2y)^5 = (3x^2y)(3x^2y)(3x^2y)(3x^2y)(3x^2y)$

$\left(\frac{2}{3}\right)^6 = \underbrace{\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\cdots\frac{2}{3}}_{6 \text{ times}}$

② $x^1 = x$

ex: $5^1 = 5$, $(-7)^1 = -7$, $(6x^3)^1 = 6x^3$

$(-4x^4y^6)^1 = -4x^4y^6$, $\left(\frac{3x}{5y^2}\right)^1 = \frac{3x}{5y^2}$

③ $x^0 = 1$, $x \neq 0$

ex: $5^0 = 1$, $(-7)^0 = 1$, $(6x^3)^0 = 1$; $x \neq 0$

$\left(\frac{-5x}{11y^2}\right)^0 = 1$; $x \neq 0, y \neq 0$ $\left(\frac{23}{-4}\right)^0 = 1$

④ $x^m \cdot x^n = x^{m+n}$

ex: $x^3 \cdot x^2 = x^{3+2} = \boxed{x^5}$

$x^{12} \cdot x^7 \cdot x^{\boxed{1}} = x^{12+7+1} = \boxed{x^{20}}$

$(3x^2)^6 \cdot (3x^2)^{10} = (3x^2)^{6+10} = \boxed{(3x^2)^{16}}$

$\left(\frac{x}{5y}\right)^8 \cdot \left(\frac{x}{5y}\right)^7 = \left(\frac{x}{5y}\right)^{8+7} = \boxed{\left(\frac{x}{5y}\right)^{15}}$

⑤ $(x^m)^n = x^{m \cdot n}$

ex: $(x^3)^2 = x^{3 \cdot 2} = x^6$

$\left(x^{\frac{2}{5}}\right)^{\frac{5}{2}} = x^{\frac{2}{5} \cdot \frac{5}{2}} = x^1 = \boxed{x}$

$(x^{-4})^{-8} = x^{(-4)(-8)} = \boxed{x^{32}}$

$\left[(3x^2)^6\right]^5 = (3x^2)^{6 \cdot 5} = (3x^2)^{30}$

$\left[\left(\frac{1}{2}\right)^{10}\right]^2 = \left(\frac{1}{2}\right)^{10 \cdot 2} = \left(\frac{1}{2}\right)^{20}$

⑥ $(xy)^n = x^n y^n$

ex: $(2x)^3 = 2^3 \cdot x^3 = \boxed{8x^3}$

$(-4x^3)^3 = (-4)^3 (x^3)^3 = \boxed{-64x^9}$

$(-2x^6y^3)^5 = (-2)^5 (x^{6 \cdot 5})(y^{3 \cdot 5})$

$= \boxed{-32x^{30}y^{15}}$

$(-3x^7y^2)^4 \cdot x^2y^{12} = (-3)^4 (x^7)^4 (y^2)^4 \cdot x^2y^{12} = 81x^{28}y^8 \cdot x^2y^{12}$

$= \boxed{81x^{30}y^{20}}$

(7) $\frac{x^m}{x^n} = x^{m-n}$

ex: $\frac{x^7}{x^3} = x^{7-3} = \boxed{x^4}$

$\frac{x^{12}y^8}{x^8y^5} = x^{12-8}y^{8-5} = \boxed{x^4y^3}$

$\frac{(x^6)^5 \cdot x^4 \cdot (y^8)^2}{x^{10} \cdot (y^3)^5} = \frac{x^{30} \cdot x^4 \cdot y^{16}}{x^{10}y^{15}} = \frac{x^{34}y^{16}}{x^{10}y^{15}}$

$= x^{34-10}y^{16-15}$

$= x^{24}y^1 = \boxed{x^{24}y}$

(8) $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

ex: $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \boxed{\frac{16}{81}}$

$\left(\frac{x^3}{y^4}\right)^5 = \frac{(x^3)^5}{(y^4)^5} = \boxed{\frac{x^{15}}{y^{20}}}$

$\left(\frac{2x^6}{3y^7}\right)^3 = \frac{(2x^6)^3}{(3y^7)^3} = \frac{2^3(x^6)^3}{3^3(y^7)^3} = \boxed{\frac{8x^{18}}{27y^{21}}}$

⑨ $x^{-n} = \frac{1}{x^n}$

ex: $2^{-2} = \frac{1}{2^2} = \boxed{\frac{1}{4}}$, $x^{-7} = \boxed{\frac{1}{x^7}}$, $10^{-1} = \frac{1}{10^1} = \boxed{\frac{1}{10}}$

$(x^6)^{-2} = x^{-12} = \boxed{\frac{1}{x^{12}}}$

$(x^2)^{-4} \cdot x^{-7} = x^{-8} \cdot x^{-7} = x^{-15} = \boxed{\frac{1}{x^{15}}}$

$\left(x^{-\frac{3}{2}}\right)^{\frac{2}{3}} = x^{-\frac{3}{2}\cdot\frac{2}{3}} = x^{-1} = \frac{1}{x^1} = \boxed{\frac{1}{x}}$

⑩ $\frac{x^{-n}}{y^{-m}} = \frac{y^m}{x^n}$

ex: $\frac{3^{-2}}{2^{-4}} = \frac{2^4}{3^2} = \boxed{\frac{16}{9}}$, $\frac{x^{-7}}{x^3 y^{-5}} = \frac{y^5}{x^3 x^7} = \boxed{\frac{y^5}{x^{10}}}$

$\frac{x^{12} y^{-7}}{(x^4)^{-3} y^7} = \frac{x^{12}}{x^{-12} y^7 y^7}$

$= \frac{x^{12} x^{12}}{y^{14}} = \boxed{\frac{x^{24}}{y^{14}}}$

(11) $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^{n}$

ex: $\left(\frac{5}{2}\right)^{-2} = \left(\frac{2}{5}\right)^{2} = \frac{2^2}{5^2} = \boxed{\frac{4}{25}}$

$\left(\frac{3x}{y^2}\right)^{-3} = \left(\frac{y^2}{3x}\right)^{3} = \frac{(y^2)^3}{3^3 x^3} = \boxed{\frac{y^6}{27x^3}}$

$\left(\frac{x^{-4}}{y^{-3}z^2}\right)^{-5} = \left(\frac{y^3}{x^4 z^2}\right)^{-5} = \left(\frac{x^4 z^2}{y^3}\right)^{5} = \boxed{\frac{x^{20} z^{10}}{y^{15}}}$

Distribute

① $4(2x^2 - 5x + 1) = 4\cdot 2x^2 - 4\cdot 5x + 4\cdot 1$

$= \boxed{8x^2 - 20x + 4}$

② $3x(5x^2 - 2x - 1) = 3x\cdot 5x^2 - 3x\cdot 2x - 3x\cdot 1$

$= \boxed{15x^3 - 6x^2 - 3x}$

③ $-4x^2(3x^4 + 5x^3)$

$= -4x^2\cdot 3x^4 - 4x^2\cdot 5x^3$

$= \boxed{-12x^6 - 20x^5}$

Find the area

$A = LW$

$2x^2y^3$

$5x^3y^7$

$A = 5x^3y^7 \cdot 2x^2y^3$

$= 10x^{3+2}y^{7+3}$

$A = 10x^5y^{10}$

$A = s^2$

$4x^6y^{10}$

$A = (4x^6y^{10})^2$

$A = 16x^{12}y^{20}$

Distribute & Simplify:

① $-5x^2(3x + 4) + 15x^3 - 20x^2$

$= -15x^3 - 20x^2 + 15x^3 - 20x^2 = \boxed{-40x^2}$

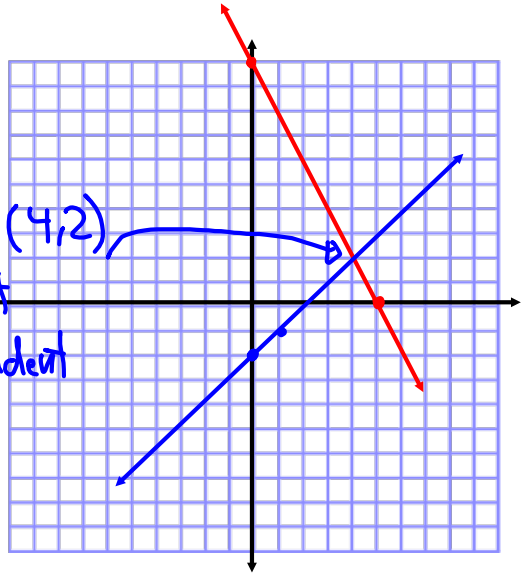
② $3xy^3(5x^2y - 4xy) - 15x^3y^4 + 12x^2y^4$

$= 15x^3y^4 - 12x^2y^4 - 15x^3y^4 + 12x^2y^4$

$= \boxed{0}$

Solve by Graphing:

$$\begin{cases} 2x + y = 10 \\ y = x - 2 \end{cases}$$



System is Consistent
Eqns are independent
Soln is (4,2)

Solve by Subs. method:

$$\begin{cases} x + 3y = 6 \\ 2x + 3y = 10 \end{cases}$$

$\Rightarrow \boxed{x = 6 - 3y}$

Hint: Isolate one of the variables.

$2(6 - 3y) + 3y = 10$

$12 - 6y + 3y = 10$

$-3y = 10 - 12$

$\boxed{y = \frac{2}{3}}$

$x = 6 - 3\left(\frac{2}{3}\right)$

$\boxed{x = 4}$

$\left(4, \frac{2}{3}\right)$

System is Consistent
Eqns are indep.

Solve

$$\begin{cases} \frac{1}{2}x - y = -3 \\ x = 6 + 2y \end{cases}$$

$\frac{1}{2}(6+2y) - y = -3$

$3 + y - y = -3$

$3 = -3$

false

No Solution

System is inconsistent

Eqns are independent.

Solve by Elimination

$$3\begin{cases} 2x - y = -2 \\ -x + 3y = -4 \end{cases} \Rightarrow \begin{cases} 6x - 3y = -6 \\ -x + 3y = -4 \end{cases}$$

$5x = -10$

$x = -2$

$-(-2) + 3y = -4$

$2 + 3y = -4$

$3y = -6$

$y = -2$

Soln. $(-2, -2)$

Solve

$$2\begin{cases} 4x - 3y = 5 \\ -8x + 6y = -10 \end{cases} \Rightarrow \begin{cases} 8x - 6y = 10 \\ -8x + 6y = -10 \end{cases}$$

$$0 = 0$$

True

Infinite # of Solns.

System is Consistent.

Eqns are dependent.

The sum of two numbers is 25. One number is 4 more than twice the other number. Find both numbers.

$$\begin{cases} x + y = 25 \\ x = 2y + 4 \end{cases}$$

$$2y + 4 + y = 25$$

$$3y = 25 - 4$$

$$3y = 21$$

$$y = 7$$

$$x = 2(7) + 4$$

$$x = 18$$

7 & 18

Kobe had 31 points on 13 made shots. 2 ptrs & 3 ptrs only. How many of each?

$x \to$ # of 2-ptrs that he made.
$y \to$ # of 3-ptrs

$$\begin{cases} x + y = 13 \\ 2x + 3y = 31 \end{cases} \quad -2$$

$$\begin{cases} -2x - 2y = -26 \\ 2x + 3y = 31 \end{cases}$$

$$y = 5$$

$$x + 5 = 13$$

$$x = 8$$

He made 8 2-ptrs, and 5 3-ptrs.

Martin has \$14.60 in quarters and nickels only.

The # of quarters is 1 fewer than twice the # of nickels. How many of each?

$x \to$ # of quarters,
$y \to$ # of nickels.

$$\begin{cases} 5x + y = 292 \\ x = 2y - 1 \end{cases} \quad \div 5$$

$$\begin{cases} 25x + 5y = 1460 \\ x = 2y - 1 \end{cases}$$

$$5(2y - 1) + y = 292$$

$$10y - 5 + y = 292$$

$$11y = 297$$

$$y = 27$$

$$x = 2(27) - 1 = 53$$

27 Nickels & 53 Quarters.

Two angles are complementary.

One of them is 4 times the other one.

Find both angles.

$$\begin{cases} x + y = 90 \\ x = 4y \end{cases}$$

$4y + y = 90$

$5y = 90$

$y = 18$

$x = 4(18)$

$x = 72$

The two angles are 18° & 72°

Two angles are supplementary.

One angle is 18° more than twice the other angle.

Find both angles.

$$\begin{cases} x + y = 180 \\ x = 2y + 18 \end{cases}$$

$2y + 18 + y = 180$

$3y = 162$

$y = 54$

$x = 2(54) + 18$

$= 108 + 18$

$x = 126$

54° & 126°

Class Quiz:

① Solve by Subs. $\begin{cases} 3x+2y=1 \\ x=y-3 \end{cases}$

$3(y-3)+2y=1$

$3y-9+2y=1$

$5y=10$ $\boxed{y=2}$

$x=2-3$ $\boxed{x=-1}$ $(-1,2)$

② Solve by addition: $\begin{cases} x-2y=8 \\ 3x+y=-4 \end{cases}$

$\begin{cases} x-2y=8 \\ 6x+2y=-8 \end{cases}$

$7x=0$

$\boxed{x=0}$

$3(0)+y=-4$

$0+y=-4$

$\boxed{y=-4}$ $(0,-4)$

Due Tomorrow:

SG 11

Lecture First

Exam: 7:30 AM